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ON CORONAL OSCILLATIONS

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ABSTRACT

Magnetoacoustic oscillations in a solar coronal inhomogeneity (e.g., coronal loop) are shown to take place with two distinct periodicities, one on an acoustic (long) time scale and the other on an Alfvénic (short) time scale. The short period modes—fast magnetoacoustic waves—are trapped in regions of low Alfvén speed; typically, this corresponds to *high* density loops or dense open field regions. Their periods may be on the order of seconds. We discuss the form of the fast oscillations for both standing modes in a closed loop and impulsively generated disturbances in a loop or open field structure. Impulsively generated waves in a density enhancement exhibit both periodic and quasi-periodic phases. Symmetric oscillations (sausage modes) are analogous to Pekeris waves in oceanography; asymmetrical (kink) disturbances are akin to Love waves in seismology. It is suggested that fast magnetoacoustic waves may explain the observed pulsations in Type IV radio events, the sausage waves providing the desired 1 s periodicities.

Magnetoacoustic oscillations provide a potentially useful diagnostic tool for determining physical conditions in the inhomogeneous corona.

Subject headings: hydromagnetics — Sun: corona

I. INTRODUCTION

The magnetic atmosphere of the solar corona can be viewed as made up of coronal loops (magnetic flux tubes), the transverse dimensions of which are much shorter than longitudinal ones. The atmosphere is dominated by magnetic forces; the flow of heat is principally along the field lines, so that lateral temperature inhomogeneities are readily maintained. Density inhomogeneities also occur. As a result, a magnetic flux tube can become "visible," standing out from its neighbors, even though the whole of the atmosphere is permeated by magnetic field.

Inhomogeneities in magnetic field strengths are probably not large, but even in a uniform field strong density variations will result in strong differences in Alfvén speed, and it is the Alfvén speed (rather than the field itself) that governs the character of oscillations.

The elastic nature of a magnetic field, and the fact that the corona is everywhere permeated by a field, suggests that the corona should support MHD oscillations. What type of magnetoacoustic oscillation can a structured, low-β, atmosphere support?

To answer such a question, we model a coronal loop by a straight magnetic flux tube (cylinder) embedded in a magnetic field. The effects of loop curvature and gravitational stratification will be ignored in order to exhibit clearly the important role of field structuring on magnetic oscillations. A coronal loop, then, is viewed as a region of density and temperature inhomogeneity in an otherwise uniform (unbounded) medium. (There may also be a corresponding inhomogeneity in magnetic field strength, but this is not important.)

There are two classes of free modes of oscillation of a magnetic tube, one with acoustic (slow mode) characteristics and the other with Alfvénic (fast mode) features. Since the Alfvén and sound speeds in the corona are generally widely separated, differing perhaps by an order of magnitude, the time scales of the two classes of oscillations are also widely separated. Additionally, the fast magnetoacoustic modes have separate time scales accordingly as the oscillations are symmetrical (sausage modes) or asymmetrical (kink modes) about the axis of the loop. The sausage modes have the shortest time scales in the system.

The fast waves arise as a free mode (with real frequency and wavenumber) only for a loop with an Alfvén speed that is lower than that in its surroundings. Thus, essentially, only *high* density loops can oscillate *freely* (without radiating energy to infinity) in the fast magnetoacoustic mode (Edwin and Roberts 1982, 1983). Dense loops may act as wave ducts, trapping fast magnetoacoustic waves (see also Habbal, Leer, and Holzer 1979).

What observational evidence is there for coronal oscillations and, in particular, for the splitting into two or more time scales described above? The solar flare and radio data seem to provide the bulk of the available evidence for coronal oscillations or pulsations. Short period oscillations, with periodicities of ~ 1 s, have long been known from radio observations (for a general review, see Krüger 1979). Indeed, it has been suggested that short period pulsations may be explained, at least in part, by considering the modulational role of an oscillating coronal flux tube (Rosenberg 1970). Also, long period (1 minute) radio pulsations have been reported by Trottet, Pick, and Heyvaerts (1979), the oscillations occurring in a large magnetic arch some 10 minutes after the rise of an active prominence. Oscillations with periods of 43 s, 80 s, and 300 s have recently been reported by Koutchmy, Zhugzhda, and Locans (1983), observing a magnetic arch in the 5303 Å green coronal line of Fe XIV, and Strauss, Kaufmann, and Opher (1980) have noted an unusual 5.6 min oscillation in a loop prominence.

We explore here the possibility of explaining this diversity of periodicities, ranging from seconds to minutes, within a single framework, namely, that of the magnetoacoustic oscillations of a flux tube. In this, we consider both *standing* modes of oscillation and *propagating* modes; in each case, the disturbances are trapped within a dense part of the corona. The standing modes may arise in a coronal loop, the ends of which are embedded in the high density chromosphere-photosphere. Propagating waves may occur in a coronal loop or in an open field region. For the most part, our discussion will center on the fast magnetoacoustic waves.

It is appropriate at this point to comment briefly on Rosenberg's (1970) theory of oscillations in a magnetic tube. His treatment of coronal oscillations is oversimplified in that it ignores the important influence of the loop's magnetic environment and considers pure radial modes only. A more thorough analysis of Rosenberg's model has been presented by Meerson, Sasarov, and Stepanov (1978), but these authors confined their attention to radiative modes, overlooking the free modes of oscillation that we exploit here. In fact, our treatment supports Rosenberg's conclusion, that oscillations with periods on the order of the tube radius divided by the fast magnetoacoustic speed may arise, though the detailed structure of such oscillations is necessarily absent in his account.

Much of the ensuing discussion of fast waves in coronal inhomogeneities simplifies considerably in the limit of a low-β plasma, appropriate for most of the solar atmosphere. In this extreme, trapped fast waves are analogous to the Love waves of seismology and the Pekeris modes of oceanography (Edwin and Roberts 1982, 1983). This unexpected analogy, valid for fast magnetoacoustic waves in the low-β limit, affords us a valuable insight in describing the nature of freely propagating modes. In particular, by analogy with extensive studies in oceanography and seismology, we are able to show that impulsively generated fast waves exhibit both periodic and quasi-periodic signatures, with periods of the order of 1 s for reasonable coronal parameters. The evolution of a disturbance generated impulsively, perhaps by a flare, is shown to resemble the pulsations recorded in the Type IV radio data. This leads us to suggest that the radio pulsations are, in fact, produced by fast magnetoacoustic waves propagating in a dense region (wave duct) of the corona.

If such an identification, of fast magnetoacoustic waves with the observed pulsations in Type IV radio events, proves correct, then the radio signature (like the seismograph signature) becomes a valuable diagnostic tool for *in situ* conditions in the corona: it allows an estimation of the Alfvén speed (both within and external to a density enhancement) and the spatial dimension of such inhomogeneities. The duration of a train of pulses is, for example, a measure of the strength of the density inhomogeneity and the distance of the observation point from the source; the periodicity is a measure of the diameter or width of the inhomogeneity; and so on.

II. MAGNETIC OSCILLATIONS

We represent a coronal loop by a straight magnetic cylinder of radius a, field strength B_0, gas density ρ_0, gas pressure p_0, and temperature T_0. Outside the cylinder, the field strength is B_e, the gas density ρ_e, the pressure p_e, and the temperature T_e (see Fig. 1a). We consider the adiabatic oscillations of this state, as described by the usual equations of ideal MHD.

The magnetoacoustic oscillations of a magnetic cylinder have been extensively investigated by Edwin and Roberts (1983), whose notation we follow. Under coronal conditions (Alfvén speeds greater than sound speeds), Edwin and Roberts show that the oscillations of the cylinder are governed by the transcendental dispersion relation (see also Wentzel 1979; Wilson 1980; Spruit 1982)

$$\rho_0(k^2v_A^2-\omega^2)m_e\frac{K_n'(m_e a)}{K_n(m_e a)}=\rho_e(k^2v_{Ae}^2-\omega^2)n_0\frac{J_n'(n_0 a)}{J_n(n_0 a)}, \quad (1)$$

where

$$m_e^2=\frac{(k^2c_e^2-\omega^2)(k^2v_{Ae}^2-\omega^2)}{(c_e^2+v_{Ae}^2)(k^2c_{Te}^2-\omega^2)},$$

$$n_0^2=\frac{(k^2c_0^2-\omega^2)(k^2v_A^2-\omega^2)}{(c_0^2+v_A^2)(\omega^2-k^2c_T^2)},$$

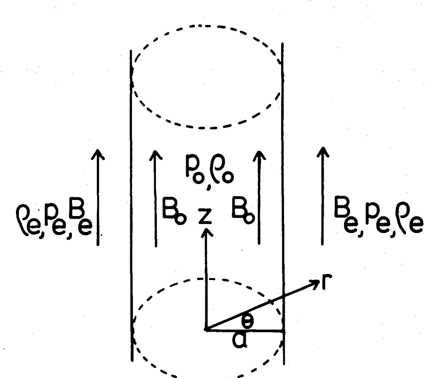

FIG. 1a

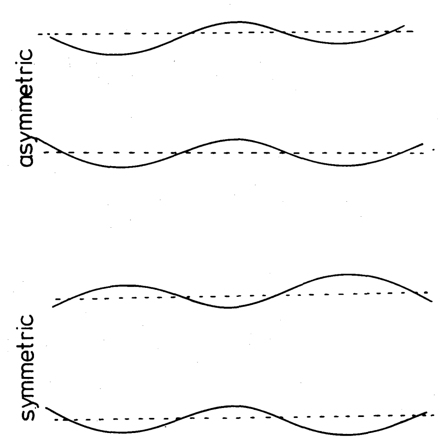

FIG. 1b

FIG. 1.—(a) The equilibrium structure and (b) modes of oscillation of a cylindrical magnetic flux tube, showing the form of symmetric (sausage) and asymmetric (kink) modes.

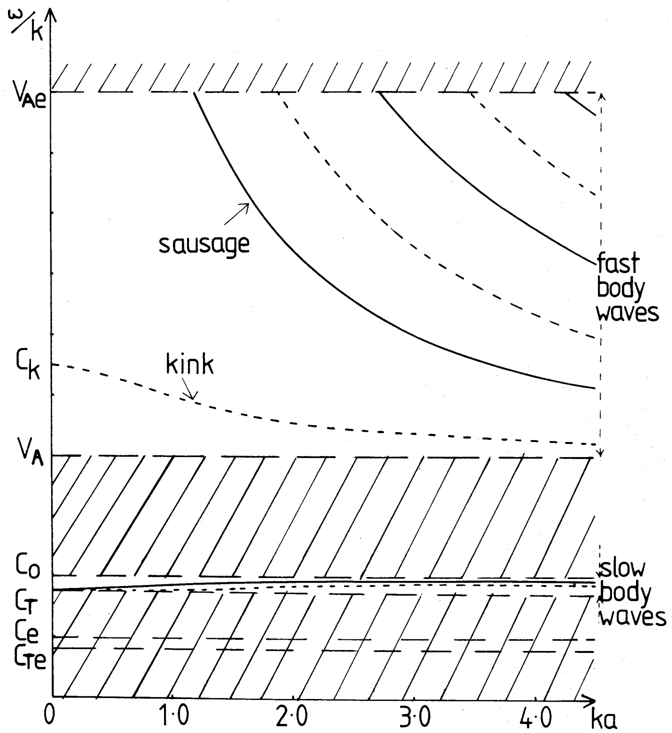

FIG. 2.—The phase-speed ω/k as a function of longitudinal wavenumber k for the fast and slow magnetoacoustic waves in a flux tube under coronal conditions, $v_{Ae} > v_A > c_0 > c_T > c_e$. *Solid curves*, sausage modes; *dashed curves*, kink modes. (After Edwin and Roberts 1983.)

and J_n, K_n are Bessel functions of order n, with derivatives J_n', K_n'. In the above, k is the wavenumber along the magnetic field, ω the frequency, and all disturbances $\boldsymbol{v}$ are assumed to have Fourier form

$$\boldsymbol{v} = \boldsymbol{v}(r) \exp [i(\omega t + n\theta - kz)]$$

in cylindrical coordinates (r, θ, z). Only the $n = 0$ (sausage) and $n = 1$ (kink) modes will be considered; their geometrical form is sketched in Figure 1*b*. The characteristic speeds that arise are: the sound speeds $c_0 = (\gamma p_0/\rho_0)^{1/2}$, $c_e = (\gamma p_e/\rho_e)^{1/2}$; the Alfvén speeds $v_A = (B_0^2/4\pi\rho_0)^{1/2}$, $v_{Ae} = (B_e^2/4\pi\rho_e)^{1/2}$; and the tube speeds $c_T = c_0 v_A/(c_0^2 + v_A^2)^{1/2}$, $c_{Te} = c_e v_{Ae}/(c_e^2 + v_{Ae}^2)^{1/2}$.

Equation (1) has been derived under the assumption that m_e^2 is positive. For ω and k real, this corresponds to there being no radial propagation outside the cylinder $r = a$: motions in the environment arise simply in response to those generated inside the cylinder. In other words, the oscillations of the magnetic flux tube are confined to the tube and penetrate only a short distance into its surroundings. These are the *free* modes of oscillation of the tube.

The solution of equation (1) is depicted in Figure 2, where the phase speed ω/k is sketched as a function of k for coronal conditions (i.e., for Alfvén speeds larger than sound speeds). It is immediately apparent that there are two, well-separated, modal classes, corresponding to the usual fast and slow magnetoacoustic waves. Considering first the *slow* waves, we see from Figure 2 that they are only weakly dispersive, that is, their phase speeds are only weakly dependent upon the wavenumber k. Indeed, since in a low-β plasma the tube speed c_T is close to the sound speed c_0, the phase speeds of both the sausage and kink modes are given, to a good approximation (provided ka is not too large), by $\omega/k = c_T$.

By contrast, the *fast* waves are *strongly dispersive*. The phase speeds of both the sausage and kink modes lie in the range v_A to v_{Ae}. These waves exist as *free* (i.e., possessing real ω and k) modes only if $v_{Ae} > v_A$; if $v_{Ae} < v_A$, the fast waves are necessarily radiative (i.e., have complex k or ω). In other words, in an inhomogeneous corona, regions of low Alfvén speed (essentially *high* density) act as wave ducts, trapping the fast magnetoacoustic modes. Furthermore, as is evident from Figure 2, the fast *sausage* waves have a propagation cutoff and exist as free modes only for sufficiently large k (typically ka of order unity or larger). More specifically, for the sausage modes, the propagation cutoffs in the wavenumber k are given by

$$k = k_c \equiv \left[\frac{(c_0^2 + v_A^2)(v_{Ae}^2 - c_T^2)}{(v_{Ae}^2 - v_A^2)(v_{Ae}^2 - c_0^2)}\right]^{1/2} \left(\frac{j_{0,s}}{a}\right), \quad s = 1, 2, 3, \ldots, \quad (2)$$

where $j_{0,s} = (2.40, \ 5.52, \ldots)$ are the zeros of the Bessel function J_0. The frequency ω at a cutoff is $k_c v_{Ae} \equiv \omega_c$.

The higher harmonics of the fast *kink* waves also possess cutoffs but, in contrast to the sausage modes, the principal kink oscillation exists for all k. In the long wavelength limit ($ka \ll 1$), the phase speed of the principal kink wave is c_k, where

$$c_k = \left(\frac{\rho_0 v_A^2 + \rho_e v_{Ae}^2}{\rho_0 + \rho_e}\right)^{1/2}$$

is a mean Alfvén speed for the inhomogeneous medium, and is well known from earlier studies (e.g., Wilson 1980; Spruit 1981).

The behavior of the fast sausage and kink waves may be most readily examined in the c_0, $c_e \ll v_A$, v_{Ae} extreme. In particular, it is known from a study of the magnetic slab (Edwin and Roberts 1982) that in this case the dispersion relation for the fast kink waves reduces to Love's equation, describing the propagation of Love waves in seismology (Love 1911). Also, the fast sausage wave in a slab is equivalent to the Pekeris mode of oceanography (Pekeris 1948). See also Ewing, Jardetzky, and Press (1957) and Brekhovskikh (1960). The propagation characteristics of fast waves in a cylinder are similar to those in a slab (Edwin and Roberts 1983), the main difference being that the principal kink wave has phase speed v_{Ae} in the long wavelength limit of a slab (compared with c_k in a cylinder). Wavenumber cutoffs and the group velocity profiles are similar in the two geometries. Evidently, then, for a low-β plasma the fast waves given by equation (1) are the cylindrical equivalents of the Love and Pekeris modes.

We consider now two applications of the above, depending upon whether the modes arise as standing waves or propagating waves.

a) Standing Waves in a Coronal Loop

Suppose that the fast and slow waves occur as standing modes in a loop of length L, the footpoints of which are firmly anchored in the high-density chromospheric atmosphere. With disturbances assumed zero at the ends ($z = 0$, L) of the loop, we may take $k = j\pi/L$ for integers $j = 1, 2, 3, \ldots$.

The periods ($= 2\pi/\omega$) of our modes are then given (in cgs units) as follows:

$$\text{slow modes: } \tau_s = \frac{2L}{jc_T} = \frac{C_s}{j} L T_0^{-1/2}\left(1 + \frac{c_0^2}{v_A^2}\right)^{1/2}, \quad C_s = 1.2 \times 10^{-4}; \quad (3a)$$

$$\text{fast kink: } \tau_f = \frac{2L}{jc_k} = \frac{4\pi^{1/2}L}{j}\left(\frac{\rho_0 + \rho_e}{B_0^2 + B_e^2}\right)^{1/2}; \quad (3b)$$

$$\text{fast sausage: } \tau_f' = \frac{2\pi a}{c_k} = 4\pi^{3/2} a\left(\frac{\rho_0 + \rho_e}{B_0^2 + B_e^2}\right)^{1/2}. \quad (3c)$$

In equations (3a, b) the integer j, which determines the number ($= j - 1$) of nodes in the oscillation along the loop, may be taken as 1 or 2, these being the modes most easy to excite; with $j = 1$ the apex ($z = \frac{1}{2}L$) of the loop is disturbed, whereas for $j = 2$ the apex is undisturbed. In (3c) we have estimated the period of the sausage mode that is first permitted as a free oscillation (satisfying $k > k_c$) by setting $\omega = kc_k$; this mode can therefore arise only if the integer j is sufficiently large (of the order of $L/\pi a$). Of course, generating such a standing mode, with a single value of j, in exclusion of neighboring modes, may be difficult. This does not imply, however, that such short periodicities cannot arise. Indeed, we show below that short periodicities occur naturally when a propagating wave is generated by an impulsive source.

b) Impulsively Generated Fast Waves

Propagating waves, rather than standing modes, will result whenever disturbances are generated impulsively. Such waves may arise in a coronal loop, if the motions have insufficient time to reflect from the far end of the loop, or in open field regions. An obvious source of such an impulsive disturbance is the flare (providing either a single or a multiple source of disturbances), but less energetic generators should not be ruled out. If the waves are generated impulsively, then the resulting disturbance may be represented as a Fourier integral over all frequencies ω, wavenumbers k. In general, a wave packet results, its overall structure being determined by the dispersive nature of the modes.

In our discussion of the behavior of impulsively generated disturbances, we will confine attention to the low-β (sound speeds very much less than Alfvén speeds) behavior of the fast sausage modes. As we noted earlier, the fast sausage modes are closely akin to the Pekeris waves in oceanography. Fortunately, the behavior of an impulsively generated Pekeris mode is well known (Pekeris 1948; see also Ewing, Jardetzky, and Press 1957; Brekhovskikh 1960), and may be applied with only minor changes to the present discussion. In Figure 3*a* we show the temporal behavior of the fast sausage mode. The disturbance is impulsively generated at the location $z = 0$. Figure 3*a* gives the behavior at a location $z = h$. The key to understanding this evolution of the disturbance lies in the behavior of the group velocity $c_g \equiv d\omega/dk$ as a function of frequency. This behavior is sketched in Figure 3*b*.

It is apparent from Figure 3*a* that there are a number of distinct phases in the evolution of a disturbance, generated at $z = 0$ and observed at $z = h$. The signal arrives at $z = h$ with a frequency $\omega_c = k_c v_{Ae}$, having taken a time $t = h/v_{Ae}$ to travel from the source at $z = 0$. This is the start of the *periodic phase*. During the periodic phase the frequency and amplitude grow slowly until, at a time h/v_A, high-frequency information arrives from the source. The result is a strong increase in amplitude with the oscillation becoming quasi-periodic. The *quasi-periodic* stage lasts until $t = h/c_g^{\min}$, where $c_g^{\min}$ is the minimum value of the group velocity. The frequency of the oscillation at this stage is $\omega^{\min}$ (see Fig. 3*b*). Thereafter, for times $t > h/c_g^{\min}$, the disturbance at $z = h$ declines rapidly in amplitude, though still oscillating with the frequency $\omega^{\min}$; this is the *decay* (or Airy) phase.

Of particular interest here are the frequencies ω_c and $\omega^{\min}$, which are representative of the frequencies in the periodic and the quasi-periodic phases, respectively. The frequency $\omega_c = k_c v_{Ae}$ follows from (2) with $s = 1$. In terms of period $\tau_c = 2\pi/\omega_c$, the low-β limit of (2) yields

$$\tau_c = \frac{2\pi a}{j_{0,1} v_{Ae}}\left(\frac{\rho_0}{\rho_e} - 1\right)^{1/2} = \frac{2\pi a}{j_{0,1} v_A}\left(1 - \frac{\rho_e}{\rho_0}\right)^{1/2}, \quad (4)$$

on noting that $\rho_0 v_A^2 \approx \rho_e v_{Ae}^2$. The period τ_c is the *largest* time scale in the impulsively generated disturbance; all other periodicities, such as the period $\tau^{\min} = 2\pi/\omega^{\min}$ of the disturbance at the end of the quasi-periodic phase, are *smaller* than

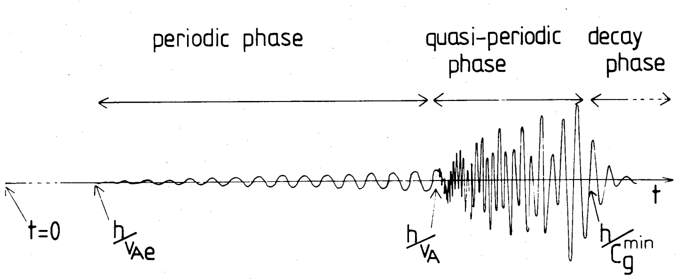

FIG. 3*a*

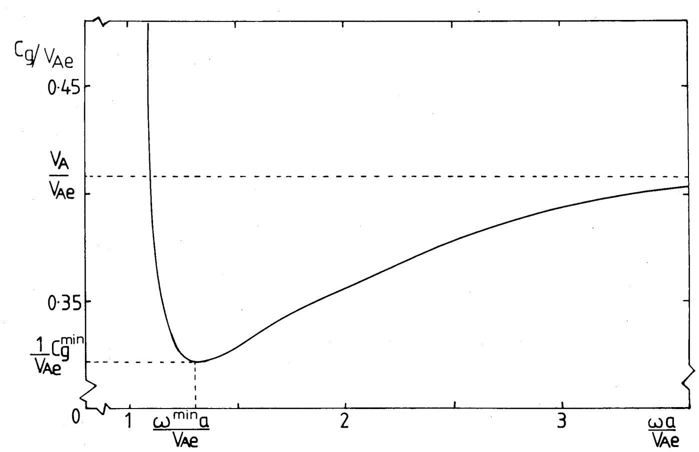

FIG. 3*b*

FIG. 3.—(*a*) A sketch of the evolution of the fast sausage wave in the low-β extreme (c_0, $c_e \ll v_A$, v_{Ae}). The sketch shows the various phases in the disturbance as recorded at an observation level $z = h$ away from an impulsive source at $z = 0$. (A similar sketch has been given by Pekeris 1948 in his discussion of waves in an ocean layer.) (*b*) The group velocity $c_g = d\omega/dk$ in units of the external Alfvén speed v_{Ae} as a function of dimensionless frequency $\omega a/v_{Ae}$ for the sausage wave in the low-β limit. The sketch is for $\rho_0/\rho_e = 6$. Notice the occurrence of a minimum in c_g at the dimensionless frequency $\omega^{\min}a/v_{Ae}$.

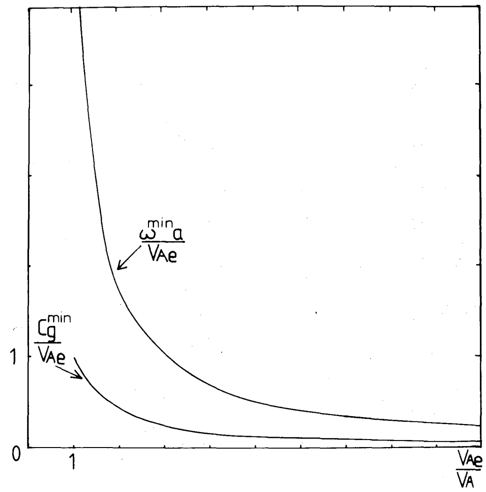

FIG. 4.—The variation of the minimum in the group velocity, and the frequency ω^{min} at which it occurs, with $v_{Ae}/v_A = (\rho_0/\rho_e)^{1/2}$ (c_g^{min} is in units of v_{Ae}, ω^{min} is in units of v_{Ae}/a).

τ_c. Note that τ_c in turn is smaller than $2\pi a/(j_{0,1}v_A) \approx 2.6(a/v_A)$, its value in a dense inhomogeneity ($\rho_0 \gg \rho_e$). Evidently, then, for typical coronal values of a and v_A, τ_c may be small, perhaps of the order of a second. We note that our formula for τ_c is similar to that given by Rosenberg (1970). The frequency ω^{min}, which characterizes the frequency at the end of the quasi-periodic phase, is sketched in Figure 4 as a function of $(\rho_0/\rho_e)^{1/2}$. Also shown in Figure 4 is the behavior of the minimum in the group velocity, c_g^{min}, as a function of $(\rho_0/\rho_e)^{1/2}$. These results will be discussed in the next section.

III. APPLICATIONS

a) Standing Waves

Consider briefly the slow modes. For coronal conditions ($c_0 \ll v_A$) we have $c_T \approx c_0$, and so with $j = 1$ equation (3a) gives $\tau_s = C_s L T_0^{-1/2}$. For $T_0 = 2 \times 10^6$ K and $L = 10^{10}$ cm, typical of high coronal loops, we obtain $\tau_s = 850$ s. Such long periodicities seem not to have been observed. A recent report of a 20 min periodicity in a giant magnetic arch (Švestka *et al.* 1982) is intriguing, but it is not associated with a loop anchored in the chromosphere (Švestka 1983, private communication).

Turning now to the fast waves, the period τ_f of the *kink* mode may be readily estimated if we suppose that the loop is very much denser than its surroundings (i.e., $\rho_0 \gg \rho_e$) and $B_0 \approx B_e$. Then, equation (3b) yields

$$\tau_f = \frac{C_f}{j}\left(\frac{LN_0^{1/2}}{B_0}\right), \quad C_f = 6.5 \times 10^{-12} \text{ cgs units}, \qquad (5)$$

for electron density N_0 inside the loop, where the field strength is B_0. For example, with $N_0 = 10^9$ cm^{-3}, $L = 10^{10}$ cm, and $B_0 = 40$ gauss we obtain $\tau_f = 50$ s for $j = 1$. To estimate the period τ'_f of the fast *sausage* wave we multiply τ_f by $\pi a/L$, giving

$$\tau'_f = C_f(\pi a N_0^{1/2}/B_0) . \qquad (6)$$

For $a/L = 10^{-2}$, τ'_f is of the order of a second.

Consider some observational examples. Trottet, Pick, and Heyvaerts (1979) have reported quasi-periodic pulsations at 169 MHz of a double source associated with a Type IV radio burst. The period was of the order of 1 min. The double structure has been interpreted as the intersections of a magnetic arch with the 169 MHz plasma level. The length of the arch was found to be of the order of 1.5×10^{10} cm, and the density has to be about 3.6×10^8 cm^{-3} or less. A spectrally resolved observation (spectrogram) of a similar case is shown in Figure 5. Seven slow, quasi-periodic pulsations separated by about 40 s occurred in the 400–800 MHz range. The fact that no drifts are observed can be interpreted as a simultaneous oscillation (free mode) of the whole loop. Presuming a high density in the magnetic arch, taking $j = 1$ or 2, and B_0 in the range from 10–30 gauss, equation (5) for the fast kink mode yields the observed period.

Another observational study of coronal oscillations that may be related to the fast kink wave is that recently described by Koutchmy, Zhugzhda, and Locans (1983). These authors report periodicities of 43 s, 80 s, and 300 s in power spectra obtained in the green coronal line 5303 Å of Fe XIV. Koutchmy *et al.* give $N_0 = 2.5 \times 10^8$ cm^{-3} and $L = 1.2 \times 10^{10}$ cm; equation (5) for the fast kink wave then gives $\tau_f = 80$ s for $j = 1$ and $B_0 = 15$ gauss. A similar interpretation, but in terms of Alfvén waves, has been given by Koutchmy *et al.*

The fast kink may also provide an explanation of the (rare) 5.6 min oscillation in a loop prominence reported by Strauss, Kaufmann, and Opher (1980). These authors give $L = 1.6 \times 10^{10}$ cm and $N_0 = 2 \times 10^{10}$ cm^{-3}, which, when combined with (5), yields the observed period if $B_0 = 44$ gauss. A similar explanation, though again in terms of Alfvén waves, is suggested by Strauss *et al.*

b) Impulsively Generated Waves

We turn now to a consideration of *short* period oscillations. The occurrence of short period oscillations in Type IV radio bursts is well known (see, for example, Rosenberg 1970; Gotwols 1972; McLean and Sheridan 1973; Achong 1974, 1976; Pick and Trottet 1978; Tapping 1978; Trottet *et al.* 1981); periodicities are typically in the range 0.5–3.0 s. Short period oscillations have also been reported in microwaves (Gaizauskas and Tapping 1980), in hard X-rays (Orwig, Frost, and Dennis 1981; Dennis, Frost, and Orwig 1981; Kiplinger *et al.* 1983), and simultaneously in hard X-rays and microwaves (Takakura *et al.* 1983; Kane *et al.* 1983). (A possible detection in X-rays has also been reported by Thomas, Davis, and Neupert 1978.)

We note also that short period oscillations are possibly superposed on the long period modulation in Figure 5. A very well observed example is shown in Figure 6. The total bandwidth is about 40 MHz. Assuming plasma emission in an isothermal, static atmosphere ($T_0 = 2 \times 10^6$ K), a vertical dimension of 23,000 km is found from the barometric equation. It is likely that the horizontal dimension, i.e., the radius of the flux tube, is even less. The apparent source diameter observed by Trottet *et al.* (1981), of some 200,000 km, therefore seems to be caused by scattering. Considering the height of 60,000 km of the 305 MHz plasma level in standard models of the density above active regions, we suggest that the observed source extends only over a small fraction of the total arch.

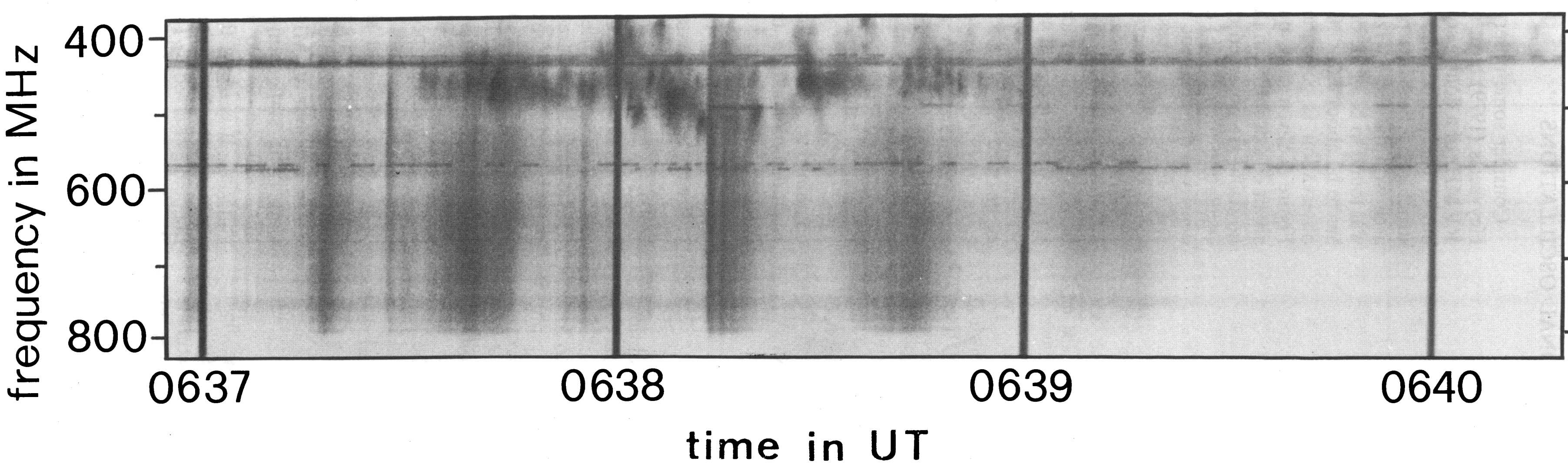

FIG. 5.—Dynamic spectrum of 1980 April 3 from the Daedalus analog spectrograph. Enhanced radio flux is shown bright. Horizontal lines are caused by terrestrial interference; vertical lines are minute marks.

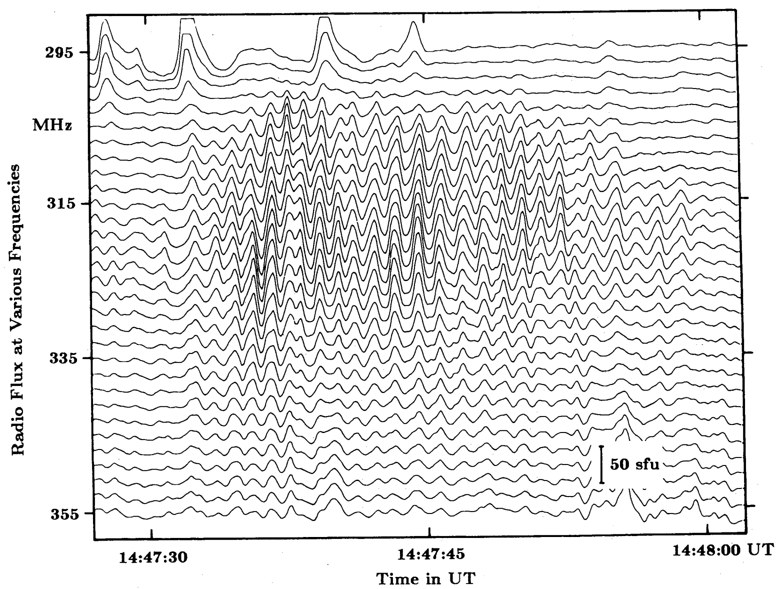

FIG. 6.—Fixed frequency plots of 1980 March 29 from the digital Icarus spectrometer. Quasi-periodic, short-period oscillations between 303 and about 343 MHz. A gliding average background (time constant 5.1 s) has been subtracted.

The occurrence of short periodicities in the coronal plasma, at radio and X-ray wavelengths, suggests (from the discussion in § II) that the fast sausage and kink modes are being generated in density enhancements. The sausage mode has the smaller time scales and also produces stronger pressure and density variations. In fact, in all cases of short (and also in long) pulsations, the Type IV emission is relatively narrow-banded and therefore, as shown by Benz and Tarnstrom (1976), likely to be radio emission by high frequency plasma waves. Kuijpers (1974) has proposed a loss-cone instability of energetic electrons for the origin of the plasma waves. Stepanov (1974) has considered a quasi-equilibrium between such an instability, diffusion of particles into the loss cone, and subsequent precipitation into the dense atmosphere at the footpoints. A free mode of oscillation of the arch in which the particles are trapped may disturb the equilibrium situation and temporarily increase the energy level of plasma waves. The sausage mode, in particular, locally changes the magnetic field strength and thus the orbits of the trapped particles. The tangent of their loss-cone angle changes by half that of the field. The equilibrium distribution is likely to be close to marginal stability of one of the "loss-cone" type plasma instabilities. Berney and Benz (1978) have shown for whistler and upper hybrid waves that the linear growth rate and stability threshold vary considerably with loss-cone (pitch) angle. The MHD oscillation moves the distribution in and out of instability. It should be noted that the final fluctuation of the radio emission may be much larger than the initial disturbance of the field strength by the wave.

When the fast sausage mode is generated impulsively, perhaps by a flare, it produces a disturbance of the form shown earlier (Figure 3a). As we have noted; there are three phases to the event: a periodic phase, a quasi-periodic phase, and a decay (Airy) phase. The *maximum* periodicity in the event is τ_c (see eq. [4]), which in turn is less than 2.6 (a/v_A). For example, with $N_0 = 10^9$ cm^{-3} and $B_0 = 40$ gauss, this *maximum* periodicity is 1 s for a tube of diameter 2×10^3 km. Our suggestion here (see also Roberts, Edwin, and Benz 1983) is that many of the reported short-period pulsations can be explained in terms of an impulsively generated fast sausage wave propagating in a dense coronal inhomogeneity. The inhomogeneity may be in an open field region, or it may be a dense loop (if reflections from its ends have not had time to modify the signal).

Because of the distinctive form of an impulsively generated disturbance, a number of comparisons between theory and observation are possible. In particular, we may compare the theoretical duration time of the quasi-periodic phase with the available observations. In Figure 7 we have sketched the duration time against the frequency attained at the end of the quasi-periodic phase. We see that the duration time of the quasi-periodic phase decreases with increasing frequency (decreasing periodicity). This may be compared directly with the results of Tapping (1978), who has plotted (see his Fig. 4) duration times against pulse repetition rates (equivalent to the reciprocal of our periodicity). The agreement between the observed correlation and the theoretical trend (shown in Fig. 7) is impressive.

In Figure 8 we have plotted the ratio of the cylinder radius a to the location height h against the duration time $\tau_{\rm dur}$ of the quasi-periodic phase. The scales in the figure are determined by the time of onset of the quasi-periodic phase $\tau_{\rm onset}$, and the pulse periodicity $\tau^{\rm min}$. These times are generally known from the observations, so Figure 8 can be used to infer the ratio a/h, and thus the scale of the inhomogeneity.

As a specific illustration of the theory, consider the event reported by McLean *et al.* (1971). These authors recorded a series of about 50 regular pulses, the period of which changed slowly from 2.5 s to 2.7 s during the 150 s lifetime of the pulse train. We do not know the onset time of the disturbance which gave rise to the pulse train—the reported start of a large flare some 25 min before the pulses began seems too

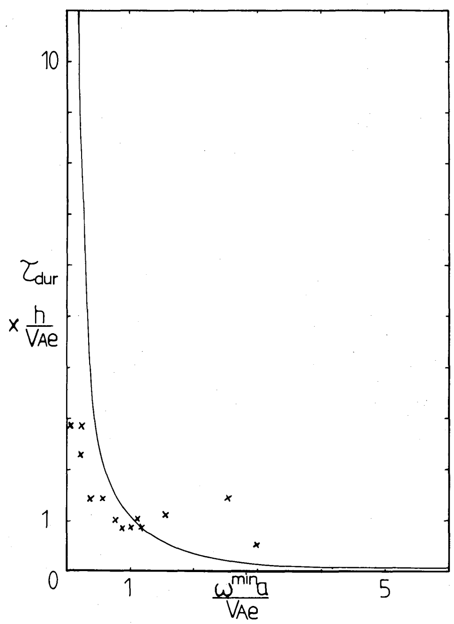

FIG. 7.—The variation of the duration time, $\tau_{dur} = h(1/c_g^{min} - 1/v_A)$, of the quasi-periodic phase with $\omega^{min}a/v_{Ae}$. The duration time is given in units of h/v_{Ae}, where h is the distance of the observation level from the source. Also shown are the data points ($\times$) from Tapping's (1978) record of meter wavelength pulsating bursts during the 1972 May 21 solar noise storm.

early to be the *direct* source of the observed pulses. If we suppose that the impulsive generation of disturbances occurred some 5 min (say) before the start of the pulses and equate this time to h/v_A, the theoretical onset time of the quasi-periodic phase, we obtain $v_A = 10^3$ km s^{-1} for $h = 3 \times 10^5$ km. The theoretical lifetime of the quasi-periodic phase is $h(1/c_g^{min} - 1/v_A)$, which, if set equal to the observed 150 s, implies that $c_g^{min} = 667$ km s^{-1}. Relation (4) implies that the observed periodicity of 2.5 s is consistent with a cylinder of diameter in excess of 1900 km. In fact, inspection of Figure 8 gives a diameter of 2250 km. All these values are reasonable, though it is difficult to be more precise without further data.

An event described by McLean and Sheridan (1973) is somewhat unusual. They observed a pulse train which started abruptly (some 7 min 40 s after a subflare) but decayed gradually. This may be reconciled with our theoretical picture (Fig. 3*a*) if we suppose, assuming that the periodic phase is lost in the noise, that the quasi-periodic phase was of fairly short duration (because c_g^{min} is close to v_A); the decay phase is then correspondingly extended. Our suggestion, then, is that the pulse train observed by McLean and Sheridan occurred within a *weakly structured* region of the corona, for which v_{Ae} is close to v_A. With only a small density difference existing between the inside and the outside of a tube, the duration of the quasi-periodic phase is reduced and the decay (Airy) phase is lengthened. A profile similar to that found by McLean and Sheridan then results.

IV. CONCLUDING REMARKS

Fast magnetoacoustic waves in dense coronal structures (loops or dense open field regions) exhibit a complex array of propagation characteristics. The waves are trapped within regions of low Alfvén speed (high gas density), propagating anisotropically in the density ducts. In the low-β conditions of the corona, the sausage and kink modes of propagation are closely akin to the Pekeris waves in the ocean and the Love waves in the Earth's crust.

A dense coronal loop, with its feet anchored in the photosphere, is able to support standing oscillations, both in the kink mode and the sausage mode. We have suggested that the kink mode may be the oscillation reported in separate events by Trottet *et al.* (1979), Strauss, Kaufmann, and Opher (1980), and Koutchmy, Zhugzhda, and Locans (1983).

An impulsively generated disturbance propagating in a dense region of the corona (either in a loop or an open field region) exhibits a complex signature in the oscillations. We have discussed the sausage mode in detail. At an observation point a distance h from the impulsive source, the disturbance begins as a low amplitude, almost constant frequency, oscillation. This *periodic phase* begins at a time h/v_{Ae} and ends at a time h/v_A, when the quasi-periodic phase begins. The *quasi-periodic phase* is of stronger amplitude and higher frequency than the earlier periodic phase; it ends at the time h/c_g^{min}, where c_g^{min} is the minimum in the group velocity. The end of the quasi-periodic phase is marked by a *decay phase*, as the amplitude of the disturbance declines (though the oscillation persists) in time.

We have argued that the above described features are similar to the observed characteristics of pulsations in Type IV radio events. In particular, we have suggested that it is the quasi-periodic phase that is most probably observed, the earlier lower amplitude (periodic) phase presumably being buried in the noise. The decay phase is generally abrupt in a high density inhomogeneity ($\rho_0 \gg \rho_e$, $v_{Ae} \gg v_A$), but becomes more protracted if the inhomogeneity is less pronounced (say, $\rho_0 = 2\rho_e$). The slowly decaying train of pulses observed by McLean and Sheridan (1973) may be an instance of such a protracted decay phase in a *weak* inhomogeneity.

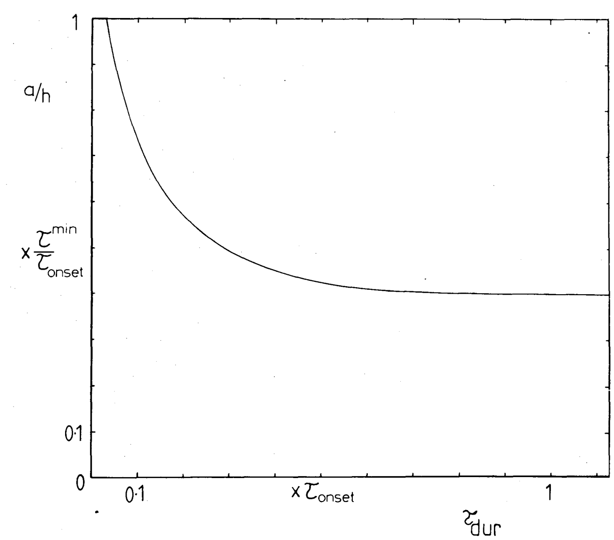

FIG. 8.—A plot of a/h (in units of τ^{min}/τ_{onset}) against τ_{dur} (in units of τ_{onset}). Here $\tau_{onset} \equiv h/v_A$ is the time that elapses between the impulsive generation of the disturbance and the onset of the quasi-periodic phase; τ^{min} is the period of the disturbance at the end of the quasi-periodic phase; τ_{dur} is the time of duration of the quasi-periodic phase. The figure may be used to deduce the width (diameter) $2a$ of the inhomogeneity in terms of the recorded values of τ_{dur}, τ_{onset}, τ^{min}, and h.

In our discussion of an impulsively generated disturbance we have considered the single impulse only. In fact, disturbances could be generated by a succession of impulses. In such a case we may anticipate that the quasi-periodic and periodic phases generated by one impulse may interact with the decay or quasi-periodic phases of a previously generated disturbance. This would produce a more complicate picture than we have indicated in Figure 3. Additionally, kink disturbances will also, in general, be generated by an impulsive source. Thus, only rarely will an event proceed in the direct and straighforward fashion described above and depicted in Figure 3. More commonly, a distortion of the various phases is to be expected due to interactions with kink waves or a repetitive generation of modes by the impulsive source.

In much of our discussion we have envisaged the impulsive source to be a flare or subflare. Indeed this is the commonly observed occurrence, a flare or subflare preceding the onset of radio pulsations. However, there are exceptions, the event reported by Gaizaukas and Tapping (1980) being a particularly clear one.

Another complicating factor in applications of our theory to the observations is the role played by dissipative processes and nonlinearities in modifying the evolution of an impulsively generated disturbance. An order-of-magnitude estimate of the radiative and conductive time scales is readily made and indicates that these time scales are long compared with the periods observed in radio pulsations. Thus, radiative and conductive effects are probably slight. Viscous effects, however, are likely to be important (Hollweg 1983) and may act as a filter preventing weakly generated disturbances from propagating far from their source. Additionally, viscous dissipation is most effective in weak field regions (see Gordon and Hollweg 1983), so it may be that only strongly (possibly shock) generated disturbances in strong field regions (say $B > 10$ gauss) are manifest far from the site at which they are produced (see also McLean *et al.* 1971). These considerations suggest that nonlinearities may be important at some stage in the evolution of a disturbance. Indeed, nonlinearities may be responsible for the observed asymmetry in the shape of individual pulses.

We conclude this paper with some brief remarks on the possibility of using radio pulsations as a diagnostic tool of physical conditions in the corona. The distinctive character of an impulsively generated disturbance, as described here for the sausage mode, makes it attractive as a diagnostic tool of *in situ* conditions in coronal inhomogeneities. The width of the inhomogeneity and the associated Alfvén speeds should be obtainable from a well-observed event, given that our identification of radio pulsations with ducted fast magnetoacoustic waves is correct. Of course, both detailed observations and theoretical calculations are necessary before such a possibility can be fully realized. Nevertheless, we feel that the potential for such a development is evident. It is in such a spirit that we offer the present discussion.

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