

PHYSICAL CONDITIONS IN INTERSTELLAR HYDROXYL AND FORMALDEHYDE CLOUDS

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SUMMARY

The physical conditions in interstellar clouds which contain the molecules OH and H₂CO have been derived from an investigation of the H, OH and H₂CO absorption spectra of Cas A, Cyg A and Tau A. The derived parameters of the typical molecule-bearing cloud are listed along with the estimates of the visual obscuration and ultra-violet radiation field. These clouds with neutral hydrogen densities in the range 2 to 70 cm⁻³ are evidently the higher density part of the distribution of interstellar clouds found in other neutral hydrogen investigations. It is shown that the lifetime of a molecule in the clouds is 10⁻⁵ to 10⁻⁴ of the lifetime of a typical cloud; this requires that the molecules should be continually formed. Most theories of OH molecule formation are inapplicable in the conditions found in these clouds. The physical parameters presented here should provide the basis for realistic theories of the formation of interstellar molecules.

1. INTRODUCTION

Both the hydroxyl radical, OH, and formaldehyde molecule, H₂CO, have a widespread distribution in the Milky Way. They are seen in absorption against large numbers of continuum radio sources on and near the galactic plane and in dense clouds at higher latitudes. This communication presents the detailed information which is now available concerning the physical conditions within the clouds which produce neutral hydrogen, OH and H₂CO absorption and proceeds to show that there is a serious problem on current molecular formation theories in producing and maintaining the observed molecular densities.

The basic material used in this investigation is the H, OH and H₂CO absorption spectra of the non-thermal sources Cas A, Cyg A and Tau A which all lie close to the galactic plane. These sources are sufficiently strong that their neutral hydrogen absorption spectra can be determined unambiguously. Since they are non-thermal there is no likelihood of anomalous (masered) OH emission confusing the OH absorption spectrum. Only in cases where the H, OH and H₂CO absorption spectra of a source can be uniquely established is it possible to make a full analysis of the physical conditions within the clouds. Furthermore, estimates are also available of another important parameter in this situation, the optical obscuration in front of each of these sources.

2. THE OBSERVATIONS

In this analysis we have used our H-line data for Cas A, Cyg A and Tau A taken with the 250 ft Mk I and 85×125 ft Mk II telescopes at Jodrell Bank and our OH data for Cas A taken with the Mk II telescope. We have supplemented it with OH data for Cyg A and Tau A from Goss (1968) and with H_2CO data for each of the sources from Zuckerman *et al.* (1970). The absorption spectrum of Cas A in each of the three lines is illustrated in Fig. 1. The spectra have been aligned in frequency using the two narrow Orion arm features.

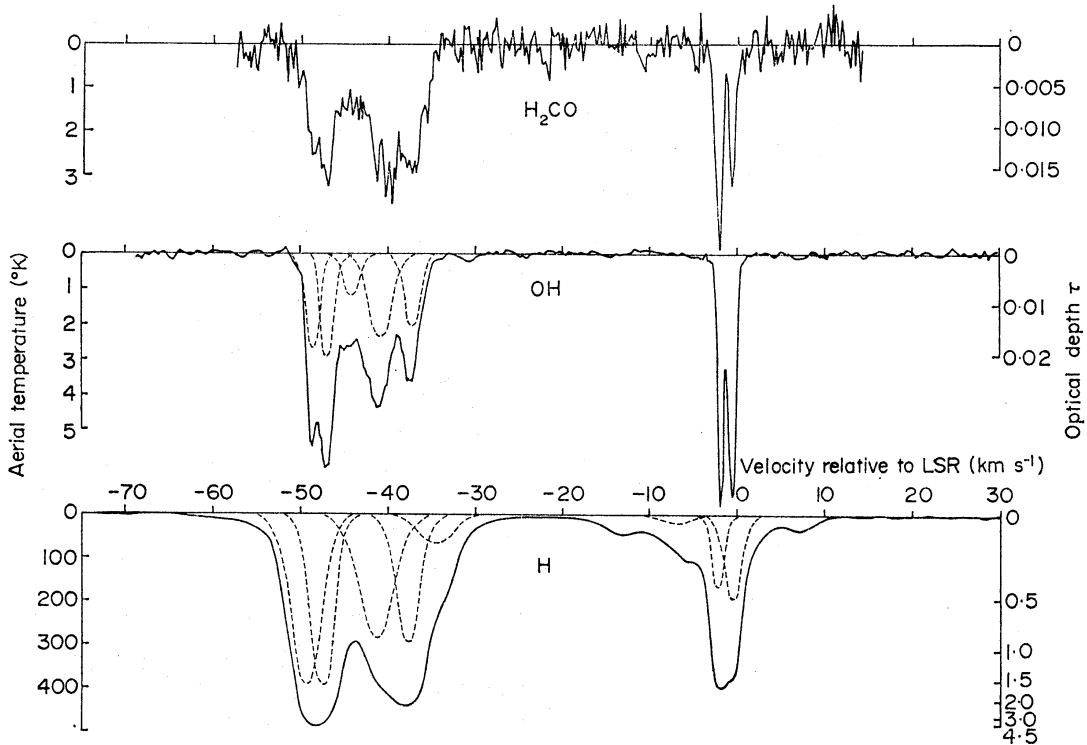


FIG. 1. The absorption spectrum of Cassiopeia A in the lines of H, OH (the 1667 MHz line of the $2\pi_{3/2}$, $J = 3/2$ state) and H_2CO (the $1_{11} \rightarrow 1_{10}$ rotational transition). The H and OH spectra were obtained at Jodrell Bank and the H_2CO spectrum is from Zuckerman *et al.* (1970). The best-fitting gaussian components of the lines of H and OH are shown; they are plotted on a linear scale in optical depth but at a reduced scale factor compared with the observed spectrum for clarity.

It can be seen from Fig. 1 that the individual clouds of highest optical depth are more clearly delineated in the spectra of the two molecules as compared with the neutral hydrogen spectrum because of the smaller thermal broadening which results from the greater mass of the molecules. These same clouds can be distinguished in the hydrogen spectra even though the individual lines are considerably broader. A striking feature of these three spectra is that the relative intensities of the lines in the H, OH, and H_2CO spectra is the same within a factor of 2 or so for the six clouds that can be readily distinguished even though they are spread over the Perseus and Orion spiral arms. Thus, clouds with similar hydrogen densities, which have presumably formed independently in different parts of two spiral arms, can produce similar densities of both OH and H_2CO molecules.

The absorption spectra can be used to provide a very useful indication of the physical conditions in the regions where molecules are abundant in the general

interstellar medium. A careful analysis of any absorption spectrum into individual components has been made using a computer program which fitted the observed spectrum to a series of gaussian profiles. The program gave a best-fitting amplitude, width and central velocity for each component.

The main components of the H and OH spectra of Cas A are shown in Fig. 1 and are listed in Table I. It can be seen that there is a good coincidence between the component velocities in the two spectra. Even in the H spectrum blend at -48 km s^{-1} where the OH spectrum shows two clear components it is possible to readily identify the same two components in the H spectrum by examining the run of the slope ($\partial T/\partial v$) of the spectrum. Clark (1965) had already noted in his interferometric study of this blend that the two sides of the blend appeared to originate in two separate positions. The OH absorption spectrum indicates the presence of a component at -44.5 km s^{-1} . Although the H data would permit the assignment of a component at this velocity, its intensity and width would be uncertain. We therefore preferred not to degrade the rest of the data by specifying an HI component at this velocity. The only effect of neglecting this component in the H-line analysis would be that the strength and width of the adjacent components would be slightly overestimated.

3. THE PHYSICAL PARAMETERS OF INDIVIDUAL CLOUDS

Estimates of the turbulent velocity and H-line kinetic temperature T_K can be made for each component following the method used by Barrett, Meeks & Weinreb (1964) who compared the H-line widths with the widths of the lines of the heavier OH molecule. Formal solutions for the r.m.s. width, σ , and T_K are given in Table I. The low kinetic temperatures ($< 100^\circ\text{K}$) found in absorption line studies previously (Shuter & Verschuur 1964; Clark 1965) are confirmed for the clearly separated H-line components. Rather higher values are suggested by the Cas A Perseus arm (-30 to -50 km s^{-1}) features. These higher values may possibly be the result of adopting a value of 4.6 for the optical depth at the centre of the -48.0 km s^{-1} complex which was the lower limit given by Shuter & Verschuur (1964) rather than the higher value of 6.7 suggested by Clark. Also, as noted above, we have not attempted to fit an H-line component corresponding to the OH component at -44.5 km s^{-1} ; this also leads to an overestimate of the temperature in the adjacent feature. If allowance is made for both these effects the kinetic temperature of the Perseus arm clouds would be reduced to approximately 100°K . Further, Verschuur (1969b) has pointed out that these clouds must have temperatures less than about 100°K or else they would be seen in emission spectra at adjacent points because of their high optical depths.

The line integral of H, OH and H_2CO densities through each of the clouds can be estimated from the optical depth integrals, $\int \tau dv$, for each absorption line component. For the evaluation of τ it is necessary to have an estimate of the line excitation temperature for H, OH and H_2CO in each cloud. The excitation temperature of neutral hydrogen is taken as the kinetic temperature, with the adopted values listed in Table I. In the case of OH and H_2CO we follow Zuckerman *et al.* (1970) who on the basis of present observational and theoretical knowledge assign an excitation temperature of 10°K for OH and 3°K for H_2CO . Both these values are probably uncertain by a factor of two and affect the derived value of the OH or H_2CO density by a similar factor.

TABLE I

Source	Velocity (km s ⁻¹)	$\int \tau_{\text{H}_2\text{CO}} dv$ (km s ⁻¹)	$\int \tau_{\text{OH}} dv$ (km s ⁻¹)	$\int \tau_{\text{H}_2\text{CO}} dv$ (km s ⁻¹)	Turbu- lence (km s ⁻¹)	Temperature Derived	Temperature Adopted	N_{H} (cm ⁻²) $\times 10^{20}$	n_{H} (cm ⁻³)	N_{OH} (cm ⁻²) $\times 10^{13}$	n_{OH} (cm ⁻³) $\times 10^{-6}$	$N_{\text{H}_2\text{CO}}$ (cm ⁻²) $\times 10^{11}$	$n_{\text{H}_2\text{CO}}$ (cm ⁻³) $\times 10^{-8}$	
Cas A	+7.6	0.30	0.0011	<0.0015	—	—	100	0.55	1.8	0.25	0.08	<2.8	<0.9	
	-0.4	2.96	0.050	0.017	0.44	99 ± 24	100	5.4	18	11.2	3.7	4.8	1.6	
	-1.9	1.75	0.040	0.019	0.31	47 ± 17	50	1.6	5.3	9.0	3.0	5.4	1.8	
	-6.6	0.65	0.0003	<0.0016	—	—	100	1.2	4.0	0.07	0.024	<2.5	<0.8	
	-12.5	0.43	<0.0015	<0.0016	—	—	100	0.80	2.7	<0.34	<0.11	<2.5	<0.8	
	-34.1	2.05	0.0023	<0.0014	—	—	100	3.8	12.5	0.52	0.17	<2.1	<0.7	
	-37.6	6.34	0.050	0.033	0.88	140 ± 30	100	11.4	37	11.2	3.7	9.2	3.1	
	-41.1	9.72	0.090	0.058	1.36	350 ± 110	100	17.7	58	20.3	6.8	16.2	5.4	
	-44.5	—	0.029	—	—	—	—	—	—	6.5	2.2	—	—	
	-47.3	8.66	0.060	0.033	0.86	139 ± 50	100	15.9	52	13.5	4.5	9.2	3.1	
	-48.8	11.2	0.057	0.016	0.40	380 ± 120	100	20.6	67	12.8	4.3	4.5	1.5	
	Tau A	+10.3	4.54	0.013	<0.002	0.91	27 ± 20	50	4.2	14	2.9	0.97	<0.53	<0.14
		+3.9	2.22	0.0074	<0.003	—	—	—	2.4	8.1	0.83	0.28	<0.81	<0.27
+2.9		1.90	<0.0018	<0.002	—	—	—	2.1	7.0	0.83	0.28	<0.70	<0.23	
-4.0		0.27	<0.0018	<0.0023	—	—	—	0.50	1.7	<0.4	<0.13	<0.89	<0.30	
Cyg A	+3.2	1.27	0.027	0.005	1.06	180 ± 40	100	2.3	7.5	6.1	2.0	1.3	0.43	
	0.0	0.82	<0.0063	<0.002	—	—	100	1.5	4.9	<1.4	<0.47	<0.53	<0.17	
	-84.5	0.99	<0.0068	<0.005	—	—	100	1.8	5.8	<1.5	<0.51	<1.34	<0.45	

Estimates can be made of the volume densities within the clouds if the cloud depth is known. Since the H clouds show a high fractional absorption it is clear that they must cover the whole source and have the appropriate dimensions transverse to the line of sight. Interferometric studies (Clark 1965) confirm this picture. In the case of the Cas A Perseus arm clouds this indicates that their diameters are 5 pc or more. A value of 10 pc is generally adopted by most authors as a working model for these particular clouds. The Cas A Orion arm clouds have minimum diameters of 2 pc. We have adopted a standard value of 10 pc for all the clouds in this discussion and the values of volume density for H, OH and H₂CO are calculated on this basis in Table I.

We are now in a position to summarize our knowledge of the physical properties of the absorbing clouds. These we believe are the best estimates currently available from modern data. The averaged parameters of the seven highest density H clouds in Table I that show molecular line absorption are listed in Table II. The average cloud with a hydrogen density of 36 cm⁻³ and a radius of 10 pc, has a mass of 640 M_⊙. In such a cloud the ratio of the molecular column density to neutral hydrogen column density is 1.1 × 10⁻⁷ for OH and 2 × 10⁻⁹ for H₂CO. Table I shows that, although there may be an uncertainty of a factor of two or so in the quoted densities, molecules are formed in clouds where the neutral hydrogen densities are typically only a few tens of atoms per cm⁻³. Furthermore at these densities hydrogen is expected to be mainly in the atomic form; Hollenbach, Werner & Salpeter (1971) calculate that only a few per cent of the mass would be in the form of molecules. Accordingly we may take it that the hydrogen densities and the cloud mass given in Table II account for essentially all the hydrogen, both atomic and molecular.

TABLE II

Mean parameters of interstellar molecule bearing absorption clouds

$\int n(\text{H}) dl = N(\text{H})$	1.1 × 10 ²¹ cm ⁻²
Temperature (H)	50–100 K
Temperature (OH)	10 K
Temperature (H ₂ CO)	3 K
N(OH)/N(H)	1.1 × 10 ⁻⁷
N(H ₂ CO)/N(H)	2 × 10 ⁻⁹
Velocity dispersion (σ)	0.74 km s ⁻¹
Cloud diameter	10 pc
Neutral hydrogen density, n(H)	36 cm ⁻³
Cloud mass	640 M _⊙
Visual absorption for cloud* (at cloud centre)	0.2 ^m –0.4 ^m
Interstellar radiation*	40 × 10 ⁻¹⁸ erg cm ⁻³ Å ⁻¹ at 1000 Å 50 × 10 ⁻¹⁸ erg cm ⁻³ Å ⁻¹ at 1400 Å 30 × 10 ⁻¹⁸ erg cm ⁻³ Å ⁻¹ at 2200 Å
Magnetic field*	2–20 μ Gauss

* See text for sources of these parameters.

Another important parameter of the absorbing clouds in the present context is the amount of obscuring matter that they contain. Estimates are available for the optical absorption in front of the three radio sources studied here. For the line of sight to the centre of Cas A, Minkowski (1968) gives an integrated absorption of 5 magnitudes while van den Bergh (1971) suggests values in the range 3 to 7

magnitudes. There are six dense neutral hydrogen clouds and about as many low density clouds in the line of sight to Cas A, so if the dust density is distributed as the gas density then the average absorption per dense neutral hydrogen cloud is $\sim 0.8^m$. The average absorption at the centre of a cloud is therefore $\sim 0.4^m$. This value would be halved if all the 10 or so clouds, independently of gas density, had the same visual absorption. In the case of Tau A (the Crab Nebula) O'Dell (1962) derives a visual absorption in the range 1^m to 2^m ; its absorption spectrum contains three neutral hydrogen features. For Cyg A Baade & Minkowski (1954) estimate an absorption of $\sim 2.1^m$; its spectrum contains at least 8 features. These two sources would indicate a similar absorption per cloud to that obtained for Cas A.

Another factor which is believed to have an important influence on the molecular density is the interstellar radiation density between $\sim 1000 \text{ \AA}$ and 3000 \AA . The values for 1000 to 2200 \AA have recently been recalculated by Habing (1968) and are included in Table II. It is evident from the derived absorption at the centre of a typical cloud that this ultra-violet radiation will penetrate the cloud with an attenuation of only about a factor of two.

Finally, measurements of the magnetic fields in neutral hydrogen clouds lying in front of Cas A and Tau A using the Zeeman splitting technique suggest line of sight components for the fields of 2–18 μ Gauss (Verschuur 1969a, 1970; Davies, Booth & Wilson 1968). The magnetic field in the surrounding interstellar medium is probably in the range 2–3 μ Gauss (see for example Davies 1968). Fields of the magnitude found in the Zeeman experiment help to stabilize interstellar clouds.

4. COMPARISON WITH OTHER INTERSTELLAR CLOUDS

At this point it should be emphasized that the absorbing clouds discussed here may not be entirely typical of the 'normal' interstellar clouds. There is clearly a range of properties of these clouds. In their study of emission clouds at intermediate latitudes Takakubo & van Woerden (1966) assign the typical member a diameter of 7 pc and a density of 14 cm^{-3} and a line of sight cloud density of 11 kpc^{-1} . Heiles (1967) in a higher angular resolution study identified several common types of clouds. He found 'cloudlets' with a diameter of 5 pc and a density of 2 atoms cm^{-3} , and 'groups of clouds' with a diameter of 30 pc and a density of $\sim 5 \text{ atoms cm}^{-3}$. These latter he claimed were similar to the clouds seen in absorption against radio sources. In addition he identified larger lower density complexes which he described as 'rifts and sheets' and 'concentrations'.

The densities of the absorbing clouds listed in Table I overlap those of the clouds described by Takakubo and van Woerden and of the 'cloudlets' and 'groups of clouds' described by Heiles. However, the largest densities in Table I exceed by a factor of 10 those derived from the emission line studies mentioned above. In addition the temperatures of the absorbing clouds are on average less than those of the emission clouds (Shuter & Verschuur 1964; Clark 1965). From the observed number of H-line absorption features in the spectra of Cas A, Cyg A and Tau A and their known line of sight distances in the Galaxy, we estimate that there are 2–4 absorbing clouds per kpc. Comparison with the data of Takakubo and van Woerden suggest that about one cloud in four is therefore an absorbing

cloud and contains detectable amounts of OH and H₂CO. This emphasizes the widespread nature of the molecular clouds with the properties listed in Table II. Further, it is interesting to note that the bulk of the galactic OH and presumably also the H₂CO is contained in the type of cloud discussed here. The two other types of OH bearing cloud, the compact masering cloud and the high density obscuring clouds, contain only a small fraction of the galactic OH. A simple calculation based on the observed numbers of these clouds suggests that they contain in total only 10^{-3} to 10^{-2} of the OH in the Milky Way.

5. MOLECULAR DENSITY AS A FUNCTION OF GAS DENSITY

The spectra in Fig. 1 show that, although the deeper hydrogen absorption features have associated OH and H₂CO features, those with optical depth $\lesssim 0.5$ have very little associated molecular absorption. This situation is presented quantitatively in Fig. 2(a) and (b) where the observed quantity $I = \int \tau dv$ of each molecular feature is plotted against the line integral for the corresponding neutral hydrogen feature. The OH data for Cas A obtained from long integration observations provide more definitive lower limits than that for Cyg A or Tau A. Also the H₂CO data is less precise than the OH data.

If we consider the part of Fig. 2(a) where $I(H)$ for neutral hydrogen is greater than 0.3 km s^{-1} it is evident that the molecular density does not increase in direct proportion to the neutral hydrogen density as would be expected for a constant molecule to hydrogen ratio. The relationship between the molecular and gas densities can be expressed in two ways. Firstly the OH data in this range of $I(H)$ can be fitted to a power law $I(\text{OH}) = \text{const. } I(H)^x$ where $x = 2.1 \pm 0.2$. Similarly

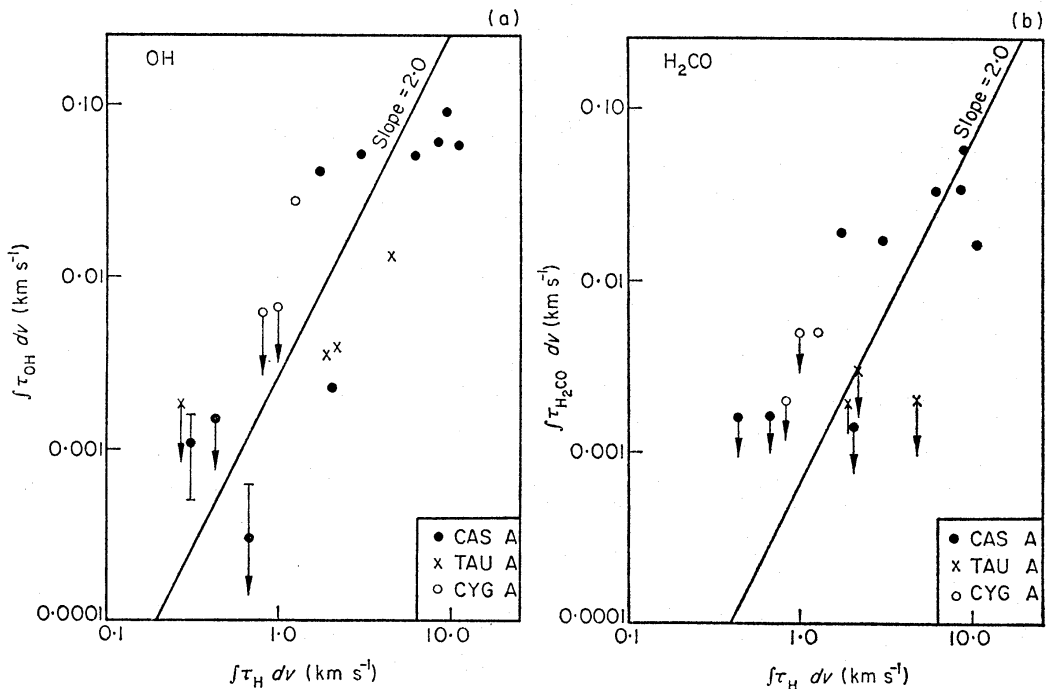


FIG. 2. Plots illustrating the dependence of the line integral, $\int \tau dv$, of molecular density upon the neutral hydrogen line integral for (a) OH and (b) H₂CO. The data used in each plot are from the absorption spectra of Cassiopeia A, Cygnus A and Taurus A.

the H_2CO data fit $x = 2.7 \pm 1.1$ although in this case the value of x is a lower limit since observational limits were used to establish it. A second and equally valid description of the data in Fig. 2(a) and 2(b) would be to say that molecules only form if the line integral of neutral hydrogen is greater than a threshold value of $I(\text{H}) > 1.0 \text{ km s}^{-1}$. Both of these descriptions suggest that the molecular density increases faster than a rate proportional to the gas density $n(\text{H})$. They are consistent, as we shall demonstrate below, with an $n(\text{H})^2$ law expected theoretically.

Our data can also be used to give quantitative estimates of the relative densities of the molecules at different neutral hydrogen densities. Whereas for the most dense HI clouds ($n(\text{H}) > 10 \text{ cm}^{-3}$) we have found

$$n(\text{OH})/n(\text{H}) = 1.1 \times 10^{-7} \text{ and } n(\text{H}_2\text{CO})/n(\text{H}) = 2 \times 10^{-9},$$

for less dense HI clouds ($n(\text{H}) < 4 \text{ cm}^{-3}$) we find $n(\text{OH})/n(\text{H}) < 1.5 \times 10^{-8}$ and $n(\text{H}_2\text{CO})/n(\text{H}) < 4 \times 10^{-10}$. We would conclude that molecules only form in detectable quantities where $n(\text{H}) \gtrsim 4 \text{ cm}^{-3}$.

6. THE TIMESCALES OF MOLECULE FORMATION

Before we consider the implications of the present results on theories of molecule formation we will discuss the relevant question of the lifetime of the interstellar clouds and the lifetime of the individual molecules. Clouds with the radius, mass and velocity dispersion indicated in Table II are readily shown not to be gravitationally bound; this conclusion has previously been drawn by Clark. They are therefore not in the gravitationally collapsing phase which might imply a short lifetime ($< 10^6$ years) but rather have a lifetime of $> 10^7$ years which is the time to double their diameters with the velocity dispersion quoted. The presence of magnetic fields within the clouds of a magnitude given in Table II would inhibit the expansion and suggest an even longer lifetime. Indeed the lifetime of any cloud will then be set by the time between cloud collision which is $\sim 10^7$ years. Thus the molecules discussed here are formed in clouds whose typical ages are half this value, namely, $\sim 0.5 \times 10^7$ years.

The time scale of molecule formation is very small compared with the typical age of an interstellar cloud as we shall now show. Using the formation of an OH molecule as an example, it can be shown that an oxygen atom moving at 1 km s^{-1} will collide with a dust grain of area $3 \times 10^{-9} \text{ cm}^2$ and form an OH molecule once every 100 years assuming 100 per cent efficiency of molecule formation and taking the physical parameters given in Table II. The lifetime of an OH molecule against a photoinduced destructive reaction in the interstellar radiation field is ~ 2000 years (Stecher & Williams 1967; McNally 1968). For H_2CO the lifetime is several orders of magnitude less than this value (Gentieu & Mentall 1970). We see that the molecular formation and destruction time scales are 10^{-5} to 10^{-4} of the lifetime of interstellar clouds. Since clouds of similar hydrogen density, but presumably having a wide range of ages up to $\sim 10^7$ years, have similar molecule density we must conclude that molecules are continually being destroyed and formed in an equilibrium situation.

7. RELEVANCE TO THEORIES OF MOLECULE FORMATION

Herbig (1963) has shown that OH formation by radiative association is unlikely because the OH molecule has not the necessary combination of excited and ground

states and accordingly we will not discuss this process further here. Carroll & Salpeter (1966) have demonstrated that OH can be formed at the observed relative densities by chemical and charge exchange reactions in gaseous regions where the kinetic temperature is about 1000°K as may occur at times of interstellar cloud collisions. This process is too slow to produce any significant amount of OH in the clouds discussed here where the temperatures are near 100°K . Moreover Carroll and Salpeter did not include the effects of dissociation which are significant at the low obscuration given in Table II; this would further reduce the OH density on their model. Stecher and Williams (1966) have proposed a mechanism in which chemical exchange reactions occur on the surface of grains. But again appreciable amounts of OH are only formed in high temperature situations as found near hot stars for example and the process is therefore not applicable to the normal interstellar clouds described here.

We are left with the surface reaction model originally proposed by McCrea & McNally (1960) and extended by McNally (1962, 1968) in which non-hydrogenic atoms collide with interstellar grains covered with a monolayer of hydrogen atoms. Whilst admitting that all the molecular reaction data were not available, McNally made a simplified calculation of the equilibrium ratio $n(\text{OH})/n(\text{H})$ which results from molecule formation on grain surfaces and the opposing photo-ionization of the molecule. A value of 1×10^{-7} is obtained for a cloud density, $n(\text{H}) = 10 \text{ cm}^{-3}$, a temperature, T , of 100°K and an interstellar radiation field similar to that given in Table II. This ratio is close to that derived for the clouds in the present study and suggests that a more rigorous theoretical investigation is now warranted using this approach.

The theory of molecule formation by surface reactions requires that the molecule density for a diatomic molecule like OH should be proportional to the product of the gas and grain space densities. If it is assumed, as is generally done, that the dust density is in turn proportional to the gas density $n(\text{H})$ then the molecule density will be proportional to $n(\text{H})^2$. Our power law fit to the OH observations gives a molecule density proportional to $n(\text{H})^{2.1 \pm 0.2}$, in agreement with this prediction.

We have discussed the situation for OH and concluded that the observations are consistent with molecule formation on dust grains. Although considerably less is known about the details of the chain of events producing H_2CO we would suggest that this molecule was also produced by surface reactions since it is formed in regions of similar physical conditions and in similar proportions relative to OH.

8. CONCLUSIONS

We have indicated that the majority (perhaps as much as 99.9 per cent) of the OH and H_2CO molecules in the Milky Way are in the type of cloud investigated here with the typical parameters listed in Table II. These clouds would appear to be at the higher gas density end of the distribution of interstellar gas clouds. An important feature of these clouds from the point of view of molecule formation is that the interstellar ultra-violet radiation field which readily ionizes and dissociates these molecules can penetrate to the centre of the molecule bearing clouds. Since the lifetime of a molecule under these conditions is a factor of 10^{-5} to 10^{-4} of the age of a cloud it must be concluded that molecules are being continually

formed. None of the published theories give an adequate description of molecule formation under the conditions prevailing in these clouds. The most promising approach would seem to be through reactions on the surface of dust grains. Our study of the physical conditions within these clouds should provide the basis for more definitive theories of molecule formation of the majority of the OH and H₂CO molecules in the Milky Way.

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